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THE ECLIPSING BINARY SIGMA AQUILAE,
THE CEPHEID VARIABLE ETA AQUILAE

By

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A. B. Park College, 1908

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THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE
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THE GRADUATE SCHOOL

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY
SUPERVISION BY Charles Clayton Wylie

ENTITLED The Eclipsing Binary Sigma Aquilae,

The Cepheid Variable Eta Aquilae.

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR
THE DEGREE OF Doctor of Philosophy

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I. THE ECLIPSING BINARY SIGMA AQUILAE.

The variable radial velocity of σ Aquilae (H. R. 7474, magnitude 5.17, spectrum B8) was discovered at Mt. Wilson in 1912, and the spectroscopic orbit was published by Jordan in 1916.¹ Jordan pointed out that the large value of K suggested an eclipsing system.

The spectra of both components appear on the plates of this star. Eclipsing binaries showing both spectra are the rare and favorable cases, since by combining the spectroscopic data with that from a good light-curve the dimensions of the system and the masses of the bodies can be found.

A short series of photometric observations was made on this star by Dr. Elmer Dershem at the University of Illinois in the summer of 1918. The reduction of these showed variation in light, which was announced by Professor Stebbins at the twenty-third meeting of the American Astronomical Society.²

A second series of observations was begun in June 1920 by Professor Stebbins and myself. This was carried to the middle of October at which time ninety-seven observations, about equally divided between the two observers, had been secured.

With two or three exceptions each of these observations is the mean of three complete sets of measures, a set consisting of two readings on the comparison star, four on the variable, and then two more on the comparison star. Throughout the series, 22 Aquilae (H. R. 7303, magnitude 5.40, spectrum A₀) was used as the

¹ Publications of the Allegheny Observatory 3, 189, 1916.

² Publications of the American Astronomical Society, 4, 115, 1919.

comparison star.

The differential correction for atmospheric extinction between σ Aquilae and 22 Aquilae was taken from tables based on those in Müller's Photometrie der Gestirne. It was less than 0.015^M from sidereal time 17.4^h to 21.0^h , and few measures were taken outside this limit. The correction to be applied was determined by multiplying the tabular value by a factor depending on the transparency of the sky, which was obtained in two ways: first, from the light effect of 22 Aquilae on the instrument, corrections for instrumental variation having been applied; and second, from the observer's estimate at the time of making the observation. The second method may seem somewhat crude, but the work at this observatory indicates that the estimate of an experienced observer deserves about the same weight as a result obtained by the first method. The factor for each observation was obtained by the two independent methods and the mean used. The two stars are of nearly the same spectral type, and as observations with large extinction have been rejected or given half weight, errors due to uncertainty in this correction should be small.

The times were reduced to the sun, and the residuals from an approximate light-curve plotted using the total atmospheric extinction on σ Aquilae as abscissae. From this plot of the residuals it was decided to reject observations with an extinction greater than 0.25^M . Those with an extinction between 0.18^M and 0.25^M were given half weight, and a few which had been marked weak for some other reason, such as smoke, were given weight one-half or three-fourths.

The center of eclipse was determined graphically, and from

the epoch of the spectroscopic observations and the 1920 photometric series a correction to the Allegheny period of $+0^d.00004$ was obtained. The available observations in 1918 were not sufficient for accurate determination of the epoch of eclipse, but a larger correction to the Allegheny period would bring them into better agreement with the 1920 series.

A plot of the residuals with the time as abscissa showed no evidence of variation in the comparison star throughout the 1920 series, but the 1918 observations were systematically high by two or three hundredths of a magnitude. Because of this, and the uncertainty in the period, the few observations of the earlier series were omitted in the formation of the normals.

By using the spectroscopic period, with the correction of $+0^d.00004$ derived above, and the photometric determination of epoch of eclipse the elements

$$\text{Minimum} = \text{J. D. } 2422486.797 + 1^d.95026 \cdot E$$

were obtained. These were adopted as final.

The photo-electric observations of σ Aquilae are given in Table I. The times were reduced to the sun, and the phases computed from the final elements given above. The difference of magnitude is the amount that σ was measured brighter than 22.

The observations of the 1920 series were then assembled according to phase and grouped into normal magnitudes. The rule was in general two observations to a normal during eclipse and three to a normal outside of eclipse. At one phase outside of eclipse two half-weight observations taken on the same night were rejected, as the others could be grouped into normals of approxi-

Observations of σ Aquilae

Date	G.M.T.	Phase	Difference of Magnitude	Weight	Date	G.M.T.	Phase	Difference of Magnitude	Weight
1918 Series (not used)					242	2514.737	0.641	^M 0.472	
						2517.711	1.665	.426	1/2
242	1783.765	1.017	^M 0.353			2521.768	1.822	.423	
	1785.759	1.061	.445			.797	1.851	.455	1/2
	1786.764	0.115	.459			2522.711	0.814	.438	
	1787.773	1.124	.470			2523.646	1.749	(.405)	
	1793.728	1.229	.470			.672	1.775	.449	1/2
	1802.771	0.520	.497			2524.679	0.832	.454	
	1804.742	0.541	.500			.733	0.886	.425	
	1805.740	1.539	.488			.750	0.903	.387	
	1806.751	0.600	.457			.767	0.920	.351	
	1813.679	1.677	.461			.782	0.935	.341	
	1817.665	1.762	.452			2525.697	1.850	.409	
	1827.644	0.040	.433			2526.651	0.854	.430	
	1828.649	1.045	.411			.668	0.871	.426	
	1837.631	0.275	.458			2528.750	1.003	.349	
	1845.611	0.454	.468			2529.681	1.934	.302	
1920 Series						2530.615	0.917	.377	1/2
						.637	0.939	.328	
	2468.810	0.017	.313			2531.614	1.916	.300	1/2
	2498.774	0.279	.474			.672	0.024	.298	
	.840	0.345	.465			2532.642	0.994	.315	
	2506.763	0.468	.480	1/2		.701	1.053	.383	
	.797	0.502	.481			.734	1.086	.438	
	2508.796	0.551	.508	1/2		.751	1.103	.442	
	2509.826	1.581	.496	1/2		2533.615	0.017	(.267)	

TABLE I (Contd.)

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Date	G.M.T.	Phase	Difference of Magnitude	Weight	Date	G.M.T.	Phase	Difference of Magnitude	Weight
242	2533.639	0 ^d .031	(0 ^M .274)		242	2561.706	0 ^d .803	0 ^M .417	3/4
	.647	0.049	.298	1/2		2566.690	1.887	.325	3/4
	.661	0.063	.328	3/4		.708	1.905	.349	3/4
	.681	0.083	.415			2567.680	0.926	.377	
	.692	0.094	.402			.697	0.943	.328	1/2
	.734	0.136	.432			2570.619	1.915	.315	
	.744	0.146	.425			.638	1.934	.287	
2534.603	1.005	.329	1/2		2574.614	0.059	.335		
	.619	1.021	.342	1/2		.634	0.079	.337	1/2
	.633	1.035	.354			2583.630	1.273	.440	
	.694	1.096	.424			.645	1.288	.452	
	.710	1.112	.451			2584.600	0.293	.453	
	.724	1.126	.448			.614	0.307	.455	
2537.710	0.211	.475	3/4		2585.606	1.299	.452		
2539.609	0.160	.430				.622	1.315	.448	
	.624	0.175	.433			2587.601	1.344	.436	
2540.640	1.191	.455				.615	1.358	.431	
	.658	1.209	.440			2588.592	0.384	.451	
	.711	1.262	.457			.605	0.397	.461	
	.722	1.273	.449			2589.574	1.366	.475	
2533.690	0.589	.485	1/2			.594	1.386	.446	
2554.647	1.546	.457				2593.594	1.485	(.432)	
	.662	1.561	.465			.608	1.499	(.424)	
2559.690	0.738	.448				2595.595	1.536	.473	
	.699	0.747	.447			.608	1.549	.455	
2561.694	0.791	.424	3/4			2596.554	0.544	.462	

TABLE I (Contd.)

Date	G.M.T.	Phase	Difference or Magnitude	Weight
242	2596.566	0. ^d 556	0. ^M 460	
	2600.581	0.671	.441	
	2603.577	1.716	.430	
	.587	1.726	.423	
	2605.563	1.751	.441	
	.574	1.762	.435	
	2606.514	0.752	.440	
	2609.542	1.830	.422	
	.553	1.841	.420	1/2

mately the same value. The thirty-seven resulting normal magnitudes and the residuals from the computed light curve are given in Table II.

A plot of the normals showed a distinct ellipticity effect between minima. A periastron effect might also be suspected, but as the spectroscopic orbit is circular, this was considered error of observation. There is no appreciable reflection effect. The ellipticity factor z was obtained by Russell's graphical method, and the result used to determine the limits of the eclipses and check the definitive work which was as follows.

From normal magnitudes outside of eclipse the ellipticity effect was solved for, using the equation:

$$\underline{m}_0 - \underline{c} \cos^2 \theta = \underline{m}$$

where $\underline{m}_0$ denotes the maximum magnitude, $\underline{m}$ is the observed magnitude, θ is the phase from primary minimum, and $\underline{c}$ is a constant. Twenty-one equations for the determination of $\underline{m}_0$ and $\underline{c}$ gave

$$\underline{m}_0 = 0.^M467 \pm 0.^M0025$$

$$\underline{c} = +0.^M042 \pm 0.^M0050$$

from which $z = 0.075$.

The ellipticity effect was removed from the observations by the formula

$$\text{Rectified magnitude} = \underline{m} + 0.^M042 \cos^2 \theta$$

From a plot of the rectified magnitudes at secondary minimum the center of that eclipse was determined as phase $0.^d980$, from which $\underline{e} \cos \omega = +0.004$. The photometric observations are too few to show with certainty that the orbit is not perfectly circular. But the eccentricity of the spectroscopic orbit is given to only one

TABLE II

Normal Magnitudes

Phase	Difference of Magni- tude	Residual O-C	No. Obs.	Phase	Difference or Magni- tude	Residual O-C	No. Obs.
0. ^d 020	^M 0.305	+0. ^M 012	2	1. ^d 013	^M 0.336	-0. ^M 002	2
0.056	0.323	-0.019	2	1.044	0.368	-0.006	2
0.069	0.348	-0.017	2	1.091	0.431	+0.003	2
0.088	0.408	+0.010	2	1.114	0.447	+0.012	3
0.147	0.429	-0.004	3	1.200	0.448	+0.005	2
0.190	0.451	+0.012	2	1.268	0.453	+0.001	2
0.293	0.461	+0.009	3	1.280	0.446	-0.008	2
0.375	0.459	-0.003	3	1.319	0.445	-0.013	3
0.512	0.473	+0.006	3	1.370	0.451	-0.012	3
0.563	0.478	+0.013	3	1.543	0.462	-0.002	3
0.656	0.456	+0.000	2	1.568	0.475	+0.013	2
0.742	0.448	+0.004	2	1.710	0.426	-0.019	3
0.779	0.428	-0.011	3	1.760	0.440	+0.001	3
0.833	0.441	+0.008	3	1.826	0.422	-0.009	2
0.878	0.426	+0.005	2	1.848	0.424	+0.006	3
0.908	0.384	+0.000	2	1.896	0.337	-0.001	2
0.923	0.364	-0.002	2	1.915	0.310	+0.000	2
0.938	0.337	-0.010	2	1.934	0.294	+0.004	2
0.998	0.332	+0.005	2				

decimal, and this value of $e \cos \omega$ seems possible. The observations about secondary minimum were therefore reflected about phase 0^d.980 for further work.

From the rectified normals at both primary and secondary, elements were derived by Russell's method. It was found, however, that as Russell has pointed out¹, the value of k can be determined from partial eclipses of such small range only with the addition of data other than that derived from the light-curve. The assumption that the eclipses were grazing total and annular, with k about 0.33, gave the best representation of the light-curve. But this means a difference of over two magnitudes in the light of the components, while from the spectroscopic plates it is known to be much less. The estimate published with the spectroscopic orbit is a difference of 0.^M.5. A second independent estimate made by Jordan in April 1921 is 0.^M.3.

The second estimate was used, and from this, with $1 - \lambda_1$, and $1 - \lambda_2$ the photometric depth of minima, k , χ , and α_0 were obtained. After a few approximations the following elements were adopted as satisfactory.

$$\begin{aligned} \chi &= 1.70 & c &= 0.0799 & d &= 0.0470 \\ k &= 1.00(\text{adopted}) & 1 - \lambda_1 &= 0.121 & 1 - \lambda_2 &= 0.0905 \end{aligned}$$

$$\text{Rectified primary minimum} = 0.^M.327$$

$$\text{Rectified secondary minimum} = 0.^M.364$$

The adopted value for k corresponds to a difference of 0.^M.31 between the component stars, practically the second estimate. Using $k = 0.92$ the light-curve holds for the first estimate, 0.^M.5,

¹Astrophysical Journal, 35, 331, 1912.

so elements were computed for both values; but the masses of the component stars, and dimensions and inclination of the relative orbit, were found to be practically the same on the two assumptions, and only the results for $k = 1.00$ are given. An ephemeris was computed from the adopted elements using Blazko's tables¹, and independently by Russell's method as a check. The computed light-curve, Tables III and IV, is shown in Figure 1. The residuals from this curve are given in Table II, and from these we derive

$$P. E. \text{ Single normal} = \pm 0.0006.$$

The normal magnitudes are plotted as circles in Figure 1, the radius of each circle being equal to the computed probable error.

The data of the spectroscopic binary system², is given in in Table V. The value of $\sin i$ was known from the photometric elements. Combining this with the spectroscopic elements the absolute dimensions of the system and masses of the bodies were obtained. Following Russell³, the apparent diameter of the brighter star was computed from the magnitude and spectral type. From this and the actual diameter, the distance and parallax were obtained, the hypothetical parallax being 0".006 corresponding to a distance of about 550 light years.

The light elements are assembled in Table VI, and Figure 2 is the system represented by them. The ellipticity of the star disks has been neglected in the figure.

¹Annales de l'observatoire astronomique de Moscou, 5, 104, 1911.

²Jordan, Allegheny Publications, 3, 192, 1916.

³Publications of the Astronomical Society of the Pacific, 32, 315, 1920.

TABLE III

Constants of the Light-Curve

	Magnitude	Range	Light	Loss
Maximum	0.467^M	---	1.000	--
Primary minimum	0.285	0.182^M	0.846	0.154
Secondary minimum	0.322	0.145	0.875	0.125

Rectified Curve

Maximum	0.467^M	---	1.000	---
Primary minimum	0.327	0.140^M	0.879	0.121
Secondary minimum	0.364	0.103	0.910	0.090

TABLE IV
Light-Curve

Phase	Difference of Magnitude	Phase	Difference of Magnitude
$\pm 0.^d00$	$0.^M285$	$0.^d94$	$0.^M345$
± 0.02	0.293	0.96	0.328
± 0.04	0.316	0.98	0.322
± 0.06	0.349	1.00	0.328
± 0.08	0.384	1.02	0.345
± 0.10	0.415	1.04	0.370
± 0.116	0.430	1.06	0.396
0.15	0.434	1.08	0.419
0.20	0.440	1.096	0.430
0.30	0.453	1.20	0.443
0.40	0.464	1.30	0.457
0.488	0.467	1.40	0.466
0.60	0.462	1.463	0.467
0.70	0.450	1.50	0.467
0.80	0.437	1.60	0.459
0.864	0.430	1.70	0.447
0.88	0.418	1.80	0.434
0.90	0.395	1.834	0.430
0.92	0.369		

THE LIGHT-CURVE OF σ AQUILAE

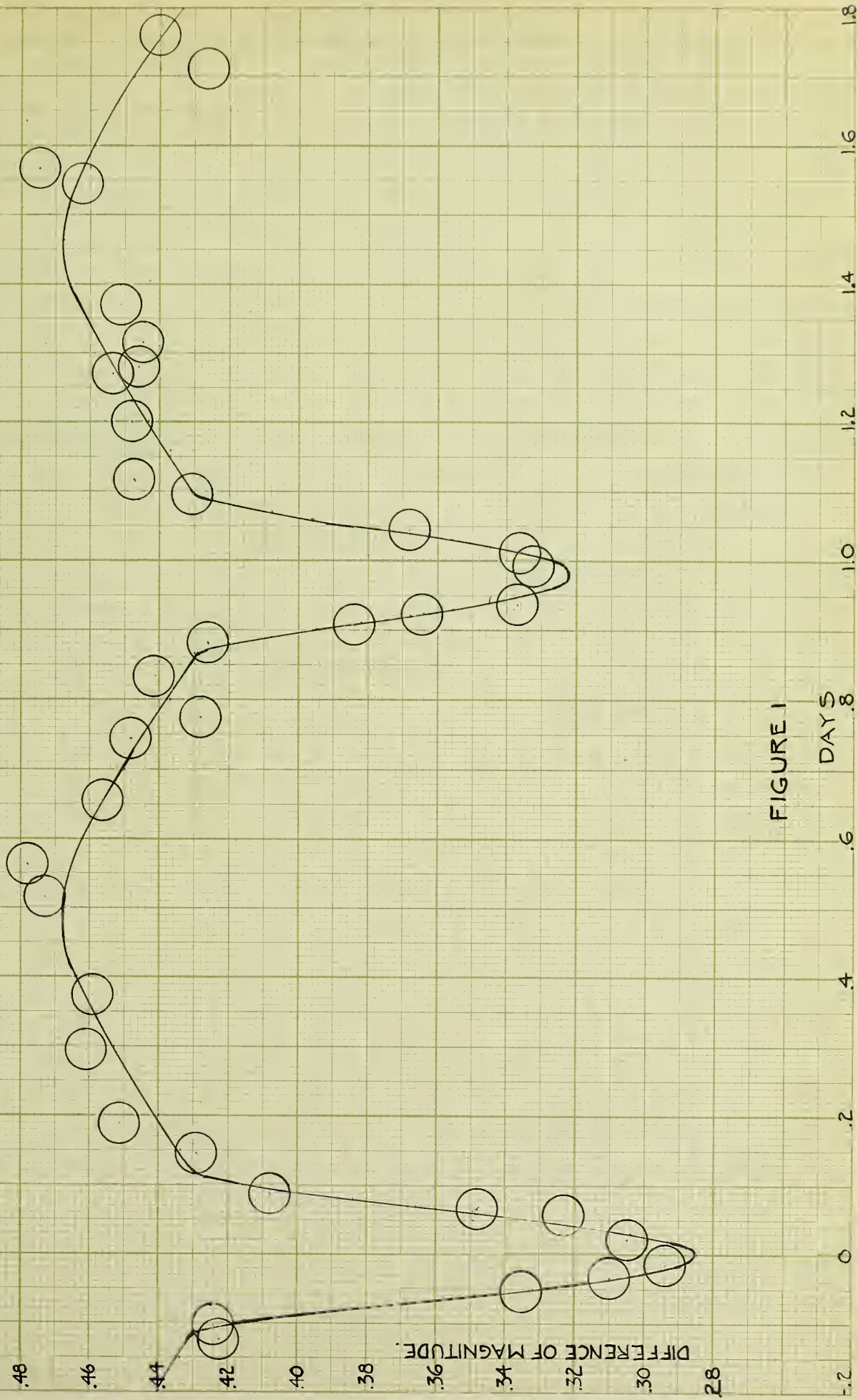


FIGURE 1

TABLE V

Spectroscopic Elements of σ Aquilae (Jordan)

$\underline{P}$	1 ^d .95022	$\pm 0^d.0001$ (estimated)
$\underline{\mu}$	184 ^o .595	$\pm 0^o.0095$
$\underline{e}$	0.0	± 0.01
$\underline{T}(\text{Max.vel.})$	J.D. 2420054.331	$\pm 0^d.0031$
$\underline{\gamma}$	-5.00 km/sec.	± 1.04 km/sec.
$\underline{K}_1$	163.52 km/sec.	± 0.35 km/sec.
$\underline{a}_1 \sin \underline{i}$	4,380,000 km	± 3600 km
$\underline{m}_1 \sin^3 \underline{i}$	5.3 $\odot$	
$\underline{K}_2$	199 km/sec.	± 4.1 km/sec.
$\underline{a}_2 \sin \underline{i}$	5,340,000 km	$\pm 110,000$ km
$\underline{m}_2 \sin^3 \underline{i}$	4.4 $\odot$	

Elements of σ Aquilae

Primary minimum, J.D. 242.		2486.797 G.M.T.
Period of revolution (Photometric).	<u>P</u>	1. ^d 95026
Phase of secondary minimum.		0. ^d 980
Semi-duration of eclipse.		0. ^d 116
Component of eccentricity	<u>e</u> cos ω	+0.004
Cosine of inclination	cos <u>i</u>	0.314
Ratio of radii.	<u>k</u>	1.00
Ratio of axes	<u>b/a</u>	0.955
Major semi-axis	<u>a</u> ₁ = <u>a</u> ₂	0.242
Minor semi-axis	<u>b</u> ₁ = <u>b</u> ₂	0.231
Light of brighter star.	<u>L</u> ₁	0.572
Light of fainter star	<u>L</u> ₂	0.428
Ratio of surface brightness	<u>J</u> ₁ / <u>J</u> ₂	1.34
Magnitude of system		5. ^M 17
Magnitude of brighter star.		5. ^M 78
Magnitude of fainter star		6. ^M 09

From Allegheny spectroscopic elements Sun = 1

Major semi-axis	<u>a</u> ₁ = <u>a</u> ₂	3.56
Minor semi-axis	<u>b</u> ₁ = <u>b</u> ₂	3.40
Density of brighter star.	ρ ₁	0.150
Density of fainter star	ρ ₂	0.125
Mean density of system.	ρ ₀	0.138
Mass of brighter star.	<u>m</u> ₁	6.19
Mass of fainter star.	<u>m</u> ₂	5.14
Parallax (from surface brightness)	π	0".006

THE SYSTEM OF σ AQUILAE.

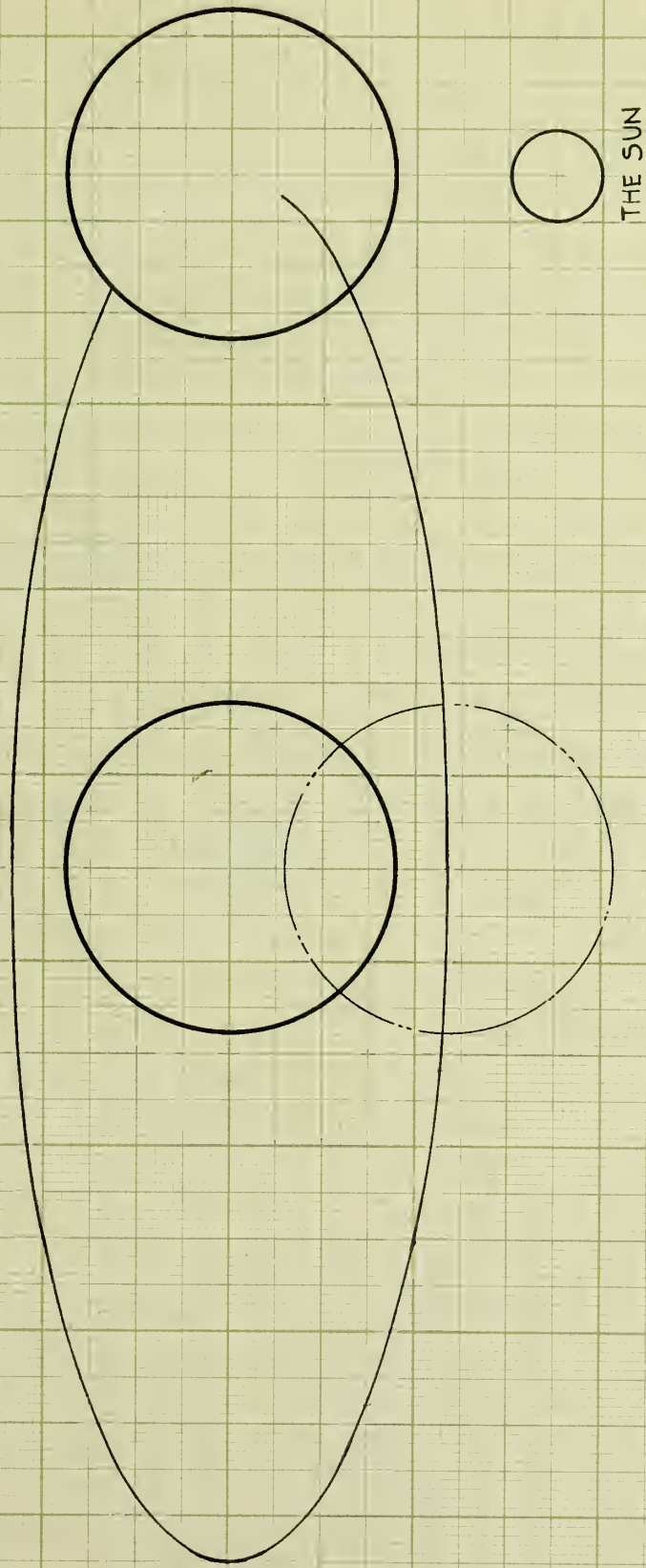


FIGURE 2.

SUMMARY

1. A series of observations made with a photo-electric photometer during 1920 gives the light-curve of σ Aquilae.
2. Using residuals from the computed curve the thirty-seven normal magnitudes show a probable error of $\pm 0.^M.006$.
3. The comparison star 22 Aquilae was constant within the limit of the measures from July to October, 1920, but a change of $0.^M.02$ between 1918 and 1920 is indicated for either 22 Aquilae or σ Aquilae.
4. Because of the small range in partial eclipse, elements of σ Aquilae from the light-curve alone would be indeterminate, but since two spectra appear on the Allegheny plates, the information on this eclipsing system is unusually complete. The elements and absolute dimensions derived are given in Table VI, and the system is shown in Figure 2.

II. THE CEPHEID VARIABLE ETA AQUILAE.

A photo-electric study of η Aquilae, one of the best known Cepheid variables, was undertaken at the University of Illinois in the hope that something of value might be contributed to the discussion of this type of variation. In particular, irregularities and secondary oscillations might be too small for certain detection by visual observers and yet be conspicuous in a photo-electric light-curve.

Observations were begun in July 1920 and carried through to November of that year. In all one hundred and eleven observations were secured, one third by Professor Stebbins and the remainder by myself. The variable and comparison stars are in the Harvard system

H. R. 7570,	η Aquilae,	magnitude 3.66-4.45,	spectrum G.
H. R. 7377,	δ Aquilae,	" 3.44	" F.
H. R. 7602,	β Aquilae,	" 3.90	" K.

The Mt. Wilson spectral types are η Aquilae F9, δ Aquilae A9, and β Aquilae G7. An observation normally consisted of thirty-six measures taken in the following order

δ 3, η 3, β 6, η 3, δ 6, η 3, β 6, η 3, δ 3,

one measure being a timing of the rate of motion of the electrometer thread between two scale divisions. Since δ Aquilae is brighter than β Aquilae, a neutral tinted shade glass was used to reduce the light to approximate equality. Throughout this paper comparisons of other stars with δ Aquilae should be interpreted as comparisons with δ Aquilae observed through the shade glass, which reduced its light about $0.\overset{M}{8}2$.

The differential correction for atmospheric extinction

between γ Aquilae and the comparison stars was taken from tables based on those in Müller's Photometrie der Gestirne. Nearly all observations were within the limits 19^h0 and 22^h5 sidereal time, during which interval the maximum tabular differential correction was for δ Aquilae, $0^M.018$, and for β Aquilae, $0^M.043$. The quantity from the table was multiplied by a factor depending on the transparency of the sky, which was obtained as in the work on σ Aquilae. The residuals from the preliminary curve were examined for a systematic hour angle correction; first, by plotting them for nights on which observations were carried on throughout nearly the entire three and a half hours during which the star could be observed; and second, by comparing residuals at different hour angles on different nights, using only the best nights. There was no indication of any appreciable systematic error remaining after the computed correction for differential extinction has been applied, although these corrections were larger than is usual in the work here.

The times were reduced to the sun, and the preliminary phases computed from the elements

$$\text{Minimum} = \text{J. D. } 2422606.562 + 7^d.176382 \cdot E$$

The period is that of Luizet¹, without his harmonic term. A plot of the observations indicated that both comparison stars are probably variable. At certain times δ Aquilae was distinctly faint, in one case as much as a tenth of a magnitude, and toward the end of the series β Aquilae was fainter than at the beginning.

In 1918 Messrs. Stebbins and Dershem used δ Aquilae as a comparison star for Nova Aquilae No. 3, and at that time it was

¹Astronomische Nachrichten, 163, 361, 1903.

suspected of variation.¹ It has been announced as a spectroscopic binary² of unknown period, and may be an eclipsing variable of small range. The other comparison star, β Aquilae, being of spectral type K, may be expected, if it varies at all, to be irregular.

As a first step toward unravelling the variations of the three stars, the observations where δ Aquilae was noticeably faint were marked. Then using only nights on which the transparency of the sky was good, and including no nights on which δ Aquilae had been marked faint at any time, the observed values of $\beta > \delta$ were plotted and a free hand curve drawn. The plotted values were compared with the curve, and one or two more rejections made on the ground that δ Aquilae was perhaps faint that night. The remainder were grouped into normal magnitudes which are given in Table II and plotted in Figure 1. From a free hand curve drawn through these normal magnitudes, the photo-electric reduction of β Aquilae to δ Aquilae could be obtained for the time of any observation.

The observed values of $\beta > \delta$ were compared with this curve, and from the residuals an unsuccessful attempt was made to get the period of variation of δ Aquilae. Phases were computed and the residuals plotted, at least around the supposed minima, for several different periods, but none was considered satisfactory. The range of variation is so small that it is probably better to await the determination of the period from spectroscopic observations.

Not knowing the variation of δ Aquilae, the following arbitrary rule was adopted. If any observed $\beta > \delta$ differed by more

¹Astrophysical Journal, 49, 354, 1919.

²Lick Observatory Bulletin, 2, 126, 1903.

Astrophysical Journal, 39, 265, 1914.

than $0.^M040$ from the curve value, all comparisons with δ Aquilae were rejected on that night. If no residual exceeded $0.^M040$, but some exceeded $0.^M030$, the observations on that night were given half weight. A few other observations had already been marked half weight because of poor observing conditions.

Observations of $\beta > \eta$ were reduced to $\delta > \eta$ by applying the correction scaled from Figure 1. Normal magnitudes for η were then formed, keeping the work on the two comparison stars separate, and a free hand curve drawn. As a check, the variation of β Aquilae was then determined by combining observed values of $\beta > \eta$ with the light-curve of η as based on δ , using only good nights. These computed values of $\beta > \delta$ as derived through η were assembled into normal magnitudes, which are plotted in Figure 1 for comparison with the values of direct observation. The satisfactory agreement shows that the progressive change between β and δ during the time of observation must be due to a variation of β , and that the light variation of η repeats itself from period to period within the limits of the present measures.

The times of maximum and minimum, and the amplitude of variation for η Aquilae were derived graphically from the normal magnitudes as follows:

Maximum,	phase	$0.^d09$,	magnitude	$-0.^M501$
Minimum,	phase	$5.^d17$,	magnitude	$-0.^M616$
Maximum - minimum		$2.^d10$,	magnitude	$1.^M12$

The correction to the time of maximum from the preliminary ephemeris is then $+0.^d09$. This was applied, giving the following elements which were adopted as final:

$$\text{Maximum} = \text{J.D. } 2422606.652 + 7.^d176382 \cdot \underline{E}$$

Table I gives the photo-electric observations of γ Aquilae. The first column gives the Greenwich mean time of observation, and the second the phase computed from the final elements; the third column gives the difference between γ Aquilae direct and δ Aquilae measured through the shade glass, and the fourth is the measured difference between γ Aquilae and β Aquilae. A positive sign means in each case that the comparison star was brighter than γ . Rejected observations are in parentheses, and half weight observations are noted under remarks. Since the average time of observation is the same for each star, the differences β brighter than δ are obtained by subtracting the third column from the fourth.

The normal magnitudes, with phases from the final elements, are given in Table III, and plotted in Figure 2, where points from the two comparison stars are kept separate, and the rejected δ Aquilae observations are plotted as crosses. No β Aquilae observations considered satisfactory at the time of making have been rejected, so all comparisons with that star are incorporated in the normal points.

The adopted light-curve of γ Aquilae is given in Table IV. It has not been drawn on Figure 2 as the observations themselves mark the course of the curve satisfactorily. Further, anyone examining suspected fluctuations will probably prefer a plot of the observations free from any evidence as to the judgment of others.

A glance at the figure shows that the reality of a considerable "hump" cannot be questioned. Since absolutely all of the observations are represented in Figure 2, it is evident that the variations of β and δ do not affect the establishment of this

TABLE I.
Observations of η Aquilae

Date, G. M. T.	Phase	Difference of Magnitude		Remarks
		δ	β	
2422523.712	3. ^d 182	(+0. ^M 282)		
.717	3.187		+0. ^M 345	
.746	3.216	(+ .255)		
.751	3.221		+ .384	
2524.706	4.176	+ .448	+ .552	
.804	4.274	+ .455	+ .566	
2525.765	5.235	+ .588	+ .680	
2526.711	6.181	+ .138	+ .262	
2528.713	1.006	- .280	- .133	δ half weight
.813	1.106	- .216	- .120	" "
2530.761	3.054	+ .211	+ .294	
2532.682	4.975	+ .606	+ .718	
2533.713	6.006	+ .270	+ .384	
.784	6.077	+ .189	+ .283	
2534.661	6.954	(- .566)	- .332	
.749	7.042	(- .536)	- .365	
.801	7.094	(- .500)	- .420	
2537.739	2.856	+ .116	+ .206	
2538.662	3.779	+ .366	+ .467	
.760	3.877	+ .377	+ .483	
2539.675	4.792	+ .566	+ .686	
.756	4.873	+ .591	+ .699	
2540.685	5.802	+ .404	+ .517	

TABLE I -(continued)

Date, G. M. T.	Phase	Difference of Magnitude		Remarks
		δ	β	
2422540.742	5 ^d .859	+0 ^M .364	+0 ^M .468	
2553.671	4.435	(+ .492)	+ .598	
.729	4.493	(+ .456)	+ .643	
2554.604	5.368	+ .563	+ .682	
.679	5.443	+ .559	+ .654	
2555.607	6.371	+ .005	+ .126	
2558.786	2.374	- .070	+ .036	
2559.717	3.305	+ .273	+ .370	
2561.731	5.319	+ .597	+ .681	
2566.622	3.032	+ .225	+ .296	
2567.599	4.009	+ .383	+ .506	δ half weight
.738	4.148	+ .393	+ .496	" "
2570.594	7.004	- .443	- .381	
.659	7.069	- .470	- .427	
2573.709	2.943	+ .106	+ .220	δ half weight
2580.585	2.642	- .008	+ .058	
.689	2.746	+ .053	+ .106	
2583.604	5.661	+ .499	+ .549	
.669	5.726	+ .471	+ .559	
.703	5.760	+ .424	+ .522	
2584.562	6.619	- .140	- .068	
.634	6.691	- .216	- .147	
.665	6.722	- .278	- .194	
.694	6.751	- .273	- .222	

TABLE I -(continued)

Date, G. M. T.	Phase	Difference of Magnitude		Remarks
		δ	β	
242585.560	0. ^d 440	-0. ^M 427	-0. ^M 363	
.586	0.466	- .410	- .359	
.634	0.514	- .414	- .340	
.649	0.529	- .389	- .328	
.664	0.544	- .377	- .315	
.679	0.559	- .390	- .322	
2586.552	1.432	(- .127)	- .045	
.599	1.479	(- .123)	- .014	
.643	1.523	(- .122)	- .008	
.685	1.565	(- .119)	- .044	
2587.560	2.440	- .029	+ .042	
2588.549	3.429	+ .302	+ .392	
.620	3.500	+ .291	+ .391	
2592.557	0.260	- .472	- .378	
.617	0.320	- .491	- .371	
.656	0.359	- .511	- .362	
.676	0.379	- .430	- .382	
2593.543	1.246		- .082	β half weight
.576	1.279	(- .158)	- .077	
.636	1.339	(- .194)	- .076	
.660	1.363	(- .151)	- .088	
2594.559	2.262	- .033	+ .048	Half weight
2596.613	4.316	+ .465	+ .553	
.628	6.331	(+ .059)	+ .081	β half weight

TABLE I -(continued)

Date, G. M. T.	Phase	Difference of Magnitude		Remarks
		δ	β	
2422598.648	6. ^d 351	(+0. ^M 040)	+0. ^M 094	
2589.615	0.141	- .469	- .417	δ half weight
.662	0.188	- .500	- .398	" "
2600.545	1.071	- .208	- .144	
.599	1.125	- .188	- .131	
.644	1.170	- .203	- .135	
2601.569	2.095	- .074	+ .012	
.590	2.116	- .032	+ .029	
.636	2.162	- .068	+ .002	
.655	2.181	- .051	- .011	
2603.604	4.130	+ .433	+ .498	
2604.633	5.159	+ .616	+ .690	
2605.526	6.051	+ .250	+ .342	
.598	6.123	+ .191	+ .254	
2606.537	7.062	- .487	- .396	
.577	7.102	- .495	- .415	
2607.547	0.896	(- .304)	- .220	
.572	0.921	(- .337)	- .206	
.589	0.938	(- .315)	- .178	
2608.544	1.893	- .042	+ .018	
.587	1.936	- .054	+ .015	
.632	1.981	- .084	- .006	Half weight
2609.581	2.930	+ .120	+ .215	
2614.545	0.718	- .346	- .286	

TABLE I -(continued)

Date, G. M. T.	Phase	Difference of Magnitude		Remarks
		δ	β	
2422615.535	1 ^d .707	(-0 ^M .055)	+0 ^M .011	
.558	1.730	(- .107)	+ .009	β half weight
2618.547	4.719	+ .566	+ .637	
.587	4.759	+ .558	+ .624	
2619.515	5.687	+ .481	+ .558	δ half weight
.558	5.730	+ .437	+ .537	" "
2626.553	5.547	+ .542	+ .609	
2627.540	6.534	- .073	- .026	
.558	6.552	- .074	- .032	
2628.542	0.360	- .450	- .367	
.565	0.383	- .476	- .392	
2632.561	4.379	+ .505	+ .558	
2633.515	5.333	+ .588	+ .648	
2641.535	6.175	+ .170	+ .210	
2642.531	7.171	- .485	- .449	
2649.494	6.958	- .422	- .387	

TABLE II

Normal Magnitudes of β Aquilae

Date	G.M.T.	Difference of Magnitude $\beta > \delta$	Number of Observations
242	2537.7	0 ^M .106	7
	2561.7	.090	4
	2582.2	.076	4
	2585.2	.066	10
	2600.7	.066	8
	2615.5	.063	1
	2633.0	.057	2

TABLE III.

Normal Magnitudes of γ Aquilae

Phase	Difference of Magnitude		No. Obs.	Phase	Difference of Magnitude		No. Obs.
	δ	$\beta(\text{reduced})$			δ	$\beta(\text{reduced})$	
0. ^d 164	-0. ^M 484	-0. ^M 474	2	3. ^d 043	+0. ^M 218	+0. ^M 197	2
0.313		- .440	3	3.204		+ .250	2
0.372	- .463		2	3.305	+ .273	+ .279	1
0.374		- .442	3	3.464	+ .296	+ .320	2
0.473	- .417	- .428	3	3.779	+ .366	+ .362	1
0.544	- .385	- .396	3	3.877	+ .377	+ .378	1
0.718	- .346	- .347	1	4.009	+ .383	+ .421	1
0.918		- .264	3	4.136	+ .420		2
1.061		- .229	3	4.139		+ .422	2
1.064	- .228		3	4.176	+ .448	+ .437	1
1.148	- .196	- .199	2	4.274	+ .455	+ .451	1
1.315		- .151	4	4.348	+ .485	+ .494	2
1.456		- .104	2	4.464		+ .526	2
1.544		- .100	2	4.739	+ .562	+ .570	2
1.715		- .050	2	4.832	+ .578	+ .588	2
1.928	- .055	- .051	3	4.975	+ .606	+ .609	1
2.124	- .058	- .052	3	5.197	+ .602	+ .596	2
2.208	- .045	- .058	2	5.326	+ .592	+ .592	2
2.407	- .050	- .044	2	5.406	+ .561	+ .574	2
2.642	- .008	- .019	1	5.547	+ .542	+ .551	1
2.746	+ .053	+ .029	1	5.674	+ .490	+ .486	2
2.856	+ .116	+ .100	1	5.727	+ .460		2
2.934	+ .115		2	5.728		+ .481	2
2.936		+ .146	2	5.760	+ .424	+ .447	1

TABLE III. -(continued)

Phase	Difference of Magnitude		No. Obs.	Phase	Difference of Magnitude		No. Obs.
	δ	$\beta(\text{reduced})$			δ	$\beta(\text{reduced})$	
5 ^d .802	+0 ^M .404	+0 ^M .413	1	6 ^d .736	-0 ^M .276	-0 ^M .283	2
5.859	+ .364	+ .364	1	6.956		- .440	2
6.006	+ .270	+ .276	1	6.958	- .422		1
6.064	+ .220	+ .226	2	7.004	- .443		1
6.123	+ .191	+ .190	1	7.023		- .468	2
6.178	+ .154	+ .152	2	7.064	- .481	- .477	2
6.355		+ .026	3	7.123		- .503	3
6.371	+ .005		1	7.136	- .490		2
6.543	- .074	- .086	2				
6.619	- .140	- .143	1				
6.691	- .216	- .222	1				

TABLE IV
Light-Curve of γ Aquilae

Phase	Difference of Magnitude	Phase	Difference of Magnitude
-2 ^d .50	+0 ^m .56	1 ^d .50	-0 ^m .11
-2.25	+ .60	1.75	- .06
-2.00	+ .61	2.00	- .05
-1.75	+ .57	2.25	- .05
-1.50	+ .49	2.50	- .03
-1.25	+ .33	2.75	+ .03
-1.00	+ .16	3.00	+ .21
-0.75	.00	3.25	+ .27
-0.50	- .20	3.50	+ .31
-0.25	- .41	3.75	+ .36
0.00	- .50	4.00	+ .40
+0.25	- .47	4.25	+ .45
0.50	- .40	4.50	+ .52
0.75	- .33	4.75	+ .57
1.00	- .24	5.00	+ .61
1.25	- .17	5.25	+ .60

THE LIGHT-CURVE OF β AQUILAE

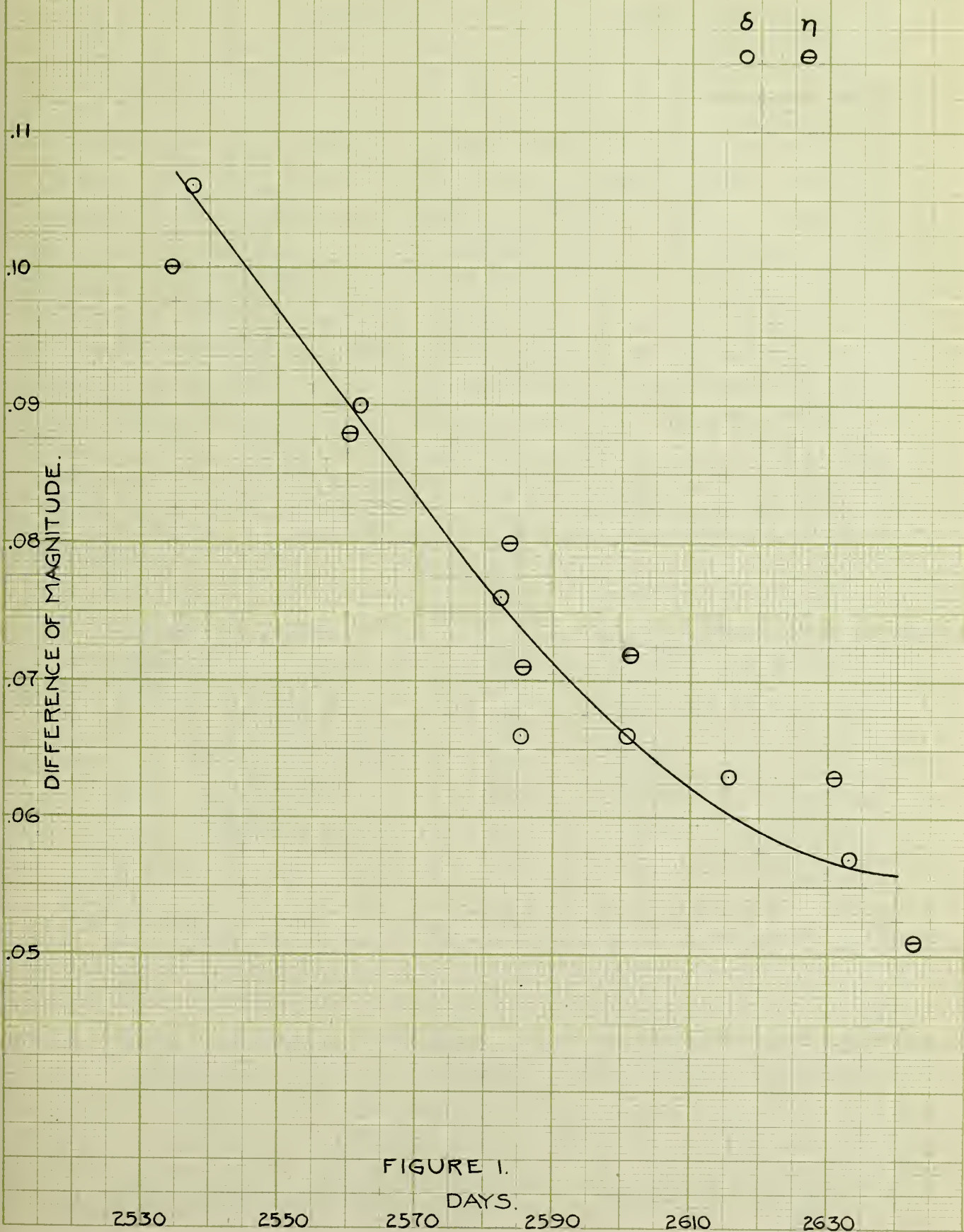


FIGURE 1.

NORMAL MAGNITUDES OF η AQUILAE

δ $\circ$ $\ominus$ $\times$
 TWO OR MORE OBSERVATIONS
 ONE OBSERVATION
 REJECTED OBSERVATIONS

DIFFERENCE OF MAGNITUDE

FIGURE 2.
DAYS

-6 -5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5 +6

-4

-3

-2

-1

0

1

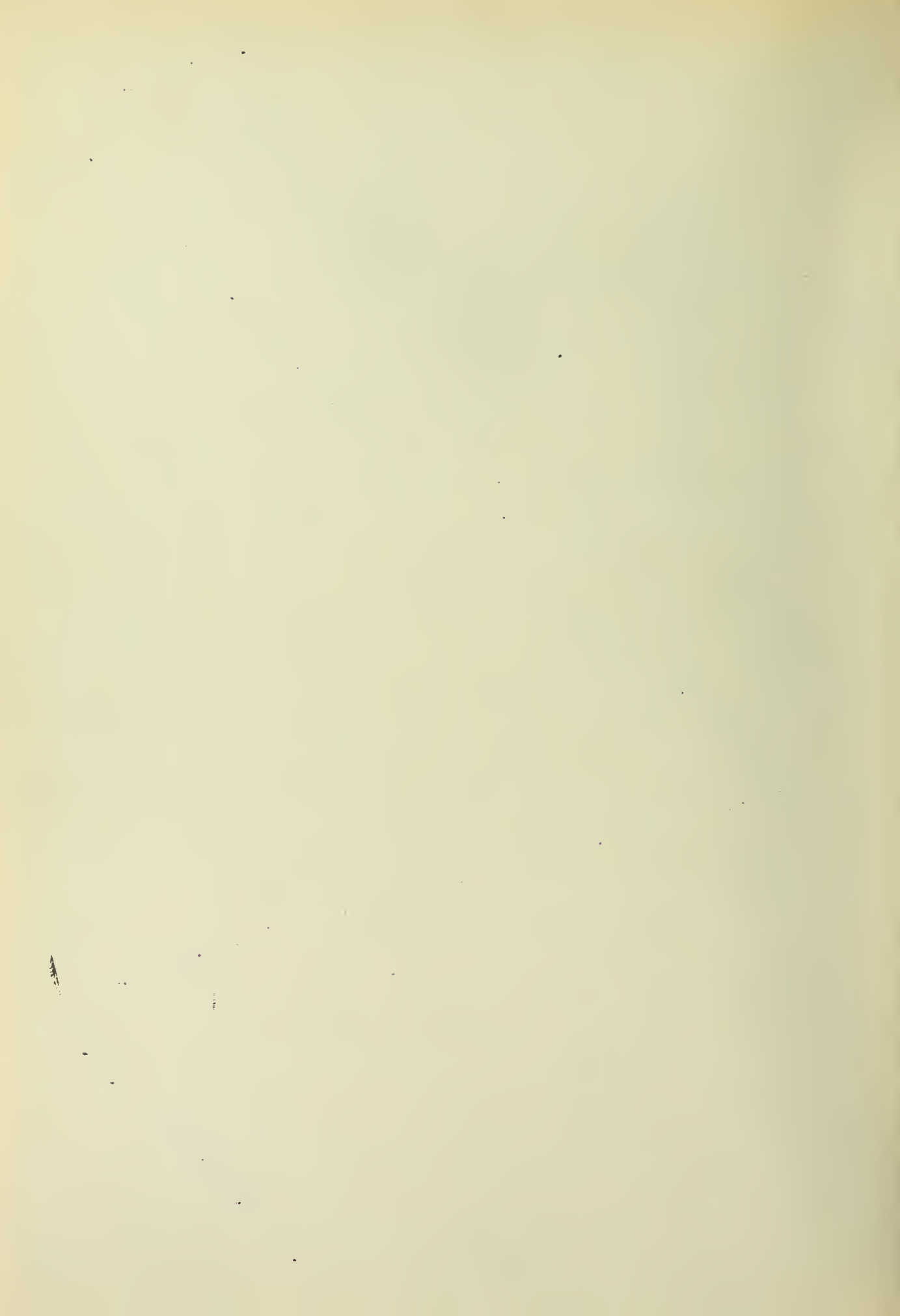
2

3

4

5

6



secondary fluctuation. Luizet's¹ description of the irregularity, "comme un arrêt dans la diminution d'éclat de la variable", or the German "Stillstand", fits the photo-electric curve. The observations indicate a practically constant magnitude for about 0.^d6 beginning at phase 1.^d8. Some of the older curves from visual observations show a pronounced increase in brightness at this point. It was considered a real "secondary maximum", but the more recent light-curves have been drawn with no irregularity in the downward slope.² The modern tendency is to regard with suspicion permanent irregularities in curves by any but the most accurate photometric methods.³ Photo-electric work has in general justified this skepticism, but in this particular instance we find confirmation of a secondary oscillation in Cepheid variation, while in addition to the pronounced secondary, there is indication of another fluctuation with maximum about phase four days. Lockyer believed the irregularities due to a secondary oscillation with period 1.^d8, one-fourth that of the principal light variation. But it is evident they cannot be represented by a superposing a simple sine curve of any period on the principal variation. Using Lockyer's sub-period we might assume that the secondary oscillation rises to its strongest maximum about phase 2.^d5. A weaker maximum is reached at phase 4.^d3, the following minimum being coincident with the principal minimum. The two succeeding secondary fluctuations are negligible.

¹Astronomische Nachrichten, 163, 364, 1903.

²Astronomische Nachrichten, 183, 265-78, 1909.

Mt. Wilson Contributions, 7, 14, 1916.

⁴Astronomische Nachrichten, 214, 195, 1921.

³Publications American Astronomical Society, 4, 265, 1921.

As a matter of some interest, the light-curve was compared with the velocity curve as determined by Wright.¹ This shows that the pronounced halt in decreasing light is at apastron on the binary hypothesis, and just before the star reaches maximum volume on the pulsation hypothesis, following Eddington.

Possible variation from epoch to epoch of the light-curves of this and other Cepheids has been discussed by previous workers.² The photo-electric observations give evidence that there was no appreciable change in the form of the curve while the observations were in progress, and the curve is not greatly different from curves previously published for this star. The maximum, however, is somewhat sharper, and the interval from minimum to maximum light is less than in other recent curves. From the curves previously referred to the values of this interval were found to be: in 1900, 2^d.38; 1906-07, 2^d.38; 1917, 2^d.20; and 1920, 2^d.10. A decrease for the more recent series is noticeable, but in all strictness comparison should be made between curves determined with the same type of photometer.

The variation of the period of γ Aquilae has also been discussed. Lockyer³, from his discussion of observations of the past century, concluded it was variable. Luizet⁴ also found a vari-

¹

Astrophysical Journal, 9, 62, 1899.

²Lockyer, Dissertation, p. 78-86, (Göttingen, 1897).

Luizet, Astronomische Nachrichten, 193, 83, 1912.

Shapley, Mt. Wilson Contributions, 6, 179, 1915.

Jordan, Publications American Astronomical Society, 4, 265, 1921.

³Lockyer, Dissertation, p. 91, (Göttingen, 1897).

⁴Luizet, Astronomische Nachrichten, 163, 361, 1903.

ation, and included a harmonic term in his value for the period, the value which has been used by most observers of γ Aquilae in recent years. On the other hand¹ some have rejected the harmonic term.

The elements are

$$\begin{aligned} \text{Maximum} &= \text{J.D. } 2396168.732 + 7^{\text{d}}.176382 \cdot \underline{E} \\ &\quad + 0.14 \sin(0^{\circ}044 \cdot \underline{E} + 304^{\circ}) \\ \text{Maximum} - \text{minimum} &= 2^{\text{d}}.373 \end{aligned}$$

Including the harmonic term the epoch of maximum derived from the photo-electric curve gives as correction to the ephemeris, $-0^{\text{d}}.006$. This is good agreement, provided we go no further, but for the minimum the correction is $+0^{\text{d}}.271$, which is not so satisfactory.

A more accurate determination of the length of the period in recent years was therefore made using the photographic observations of Kohlschütter², made in 1906-07. The photographic and photo-electric amplitudes being practically the same, comparison of the curves is direct. Using Luizet's mean period between the two series normal magnitudes were formed from the photographic observations. Three observations were combined in a normal point except on the steep portion of the curve where two were used. This steep portion, between minimum and maximum, was used in the determination of the period, as the phase can be determined with more accuracy here than at the time of maximum or minimum.

A plot showed these points nearly a quarter of a day out of phase with the photo-electric curve, a graphical determination giving as a correction $0^{\text{d}}.22$. This gives as the correction to the period $+0^{\text{d}}.00032$, from which the value for the period between 1906 and 1920 is $7^{\text{d}}.17670$

¹Hellerich, Dissertation, p 6, (Bonn, 1913).

²Astronomische Nachrichten, 183, 265, 1909.

This value for the length of the period is so much larger than the mean values of Lockyer and Luizet obtained from a discussion of observations of the past century that Luizet's tabulation of corrections to an ephemeris¹ from a constant period was brought up to date. Corrections computed from the light-curves of Kohlschütter and Lacchini,² and from the photo-electric light-curve, were added to the list. These residuals are plotted in Figure 3, points from maxima and minima being plotted separately. Normal points are formed, giving the determinations equal weight except that in the last point the photo-electric determination has been given double weight. The normal points are connected by a heavy line.

This line is distinctly concave upward, hence observational evidence indicates an increasing period at least in recent years. Certainly there is no evidence of an appreciable decrease at any time in the last 140 years. If the variation is harmonic the available portion of the curve is too little to determine either the amplitude or the period with any accuracy. However, both must be much greater than the values found by Luizet.

The question of variation of period is of importance in Cepheid theory. The pulsation hypothesis demands a shortening of the period,³ about 40 seconds annually for δ Cephei on the contraction theory of a star's energy. Similar conclusions follow on the rotation hypothesis, while according to the binary theory the

¹ Luizet, Astronomische Nachrichten, 163, 360, 1903.

² Lacchini, Astronomische Nachrichten, 214, 195, 1921.

³ Eddington, Observatory, 41, 379, 1918, 42, 339, 1919.

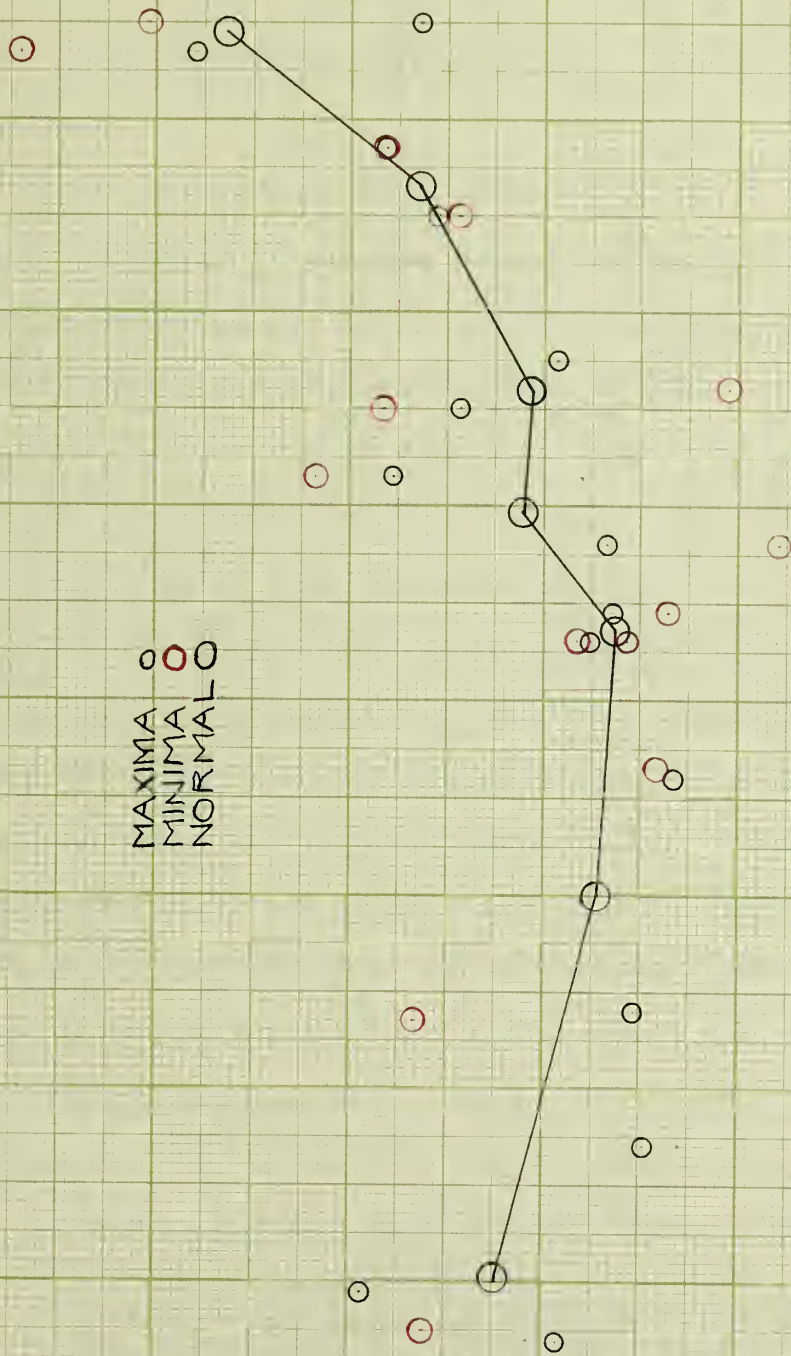
VARIATION IN THE PERIOD OF η AQUILAE

MAXIMA ○
 MINIMA ○
 NORMAL ○

DAYS
 .3
 .2
 .1
 0
 -1
 -2

YEARS
 1780 1790 1800 1810 1820 1830 1840 1850 1860 1870 1880 1890 1900 1910 1920

FIGURE 3



period may either increase or decrease.¹ Tidal action should in general lengthen the period if it is equal to or greater than the rotation period of the principal star. But if the satellite revolves in less than the rotation period, the orbital period should decrease.

There is evidence that the period of δ Cephei is decreasing.² If these suspected variations of period are accepted as real and attributed to tidal action, the deduction that η Aquilae probably rotates faster than δ Cephei follows.

Since either a rotation or a pulsation should affect the appearance of the spectral lines, the fact that the lines of η Aquilae are broader than those of δ Cephei is worthy of mention.

δ Cephei³ has the narrow well defined spectral lines typical of giant stars, while the lines of η Aquilae,⁴ also classed with the giant stars "have the general characteristics of breadth and haziness, which tend to make them objectionable for purposes of accurate measurement". In a footnote to this reference the statement is made that "There is undoubtedly a resemblance between the spectra of η Aquilae and δ Cephei, the lines of the latter being, however, better defined, - -".

¹

Plummer, Monthly Notices, 80, 507-09, 1920.

² Eddington, Observatory, 42, 338, 1919.

Ludendorff, Astronomische Nachrichten, 212, 185, 1920.

³ Adams, Observatory, 42, 167-68, 1919.

⁴ Wright, Astrophysical Journal, 9, 60, 1899.

SUMMARY

1. A series of photo-electric observations made in 1920 gives an accurate light-curve for η Aquilae.
2. The observations indicate a long period or irregular variation for β Aquilae. A progressive decrease of 0.05^M is found for an interval of three months.
3. δ Aquilae, which has been announced as a spectroscopic binary of unknown period, is suspected of being an eclipsing variable with range about 0.1^M , but no period has been found.
4. The "secondary maximum" in the light-curve of η Aquilae is confirmed as a pronounced halt, or pause, in the decrease of light. Another fluctuation, with maximum about phase four days, is indicated.
5. Within the accuracy of the measures the light-curve did not change between July and November 1920. There has perhaps been a shortening of the interval from minimum to maximum since 1900, but accurate curves determined with the same photometric equipment and at different epochs are needed for a thorough investigation of this point.
6. The period, between 1906 and 1920, is determined by comparison with Kohlschütter's photographic observations. Luizet's tabulation of residuals from the constant period $7^d.176382$ is brought up-to-date by including observations of 1906-07, 1917, and 1920. Both methods of investigation favor an increasing period at present. The evidence is decidedly against a perceptible decrease in the period at any time since 1784.
7. The pulsation hypothesis demands a shortening of the period, while according to the older binary theory, tidal action may

increase or decrease the period. Other investigators have found evidence of a decrease in the period of δ Cephei.

Either pulsation or rotation should affect the spectral lines of a star. Attention is called to the evidence that the lines of γ Aquilae are broader than those of δ Cephei, which have been examined for a pulsation effect.

ACKNOWLEDGMENT.

I am indebted to Professor Joel Stebbins for supervision and direction throughout these investigations, and for some of the observational data, and to Mr. J. K. Willy for checking some of the reductions.

VITA

Charles Clayton Wylie.

Born at Idana, Kansas, on June 18, 1886. High school education received at Marissa, Illinois. Graduated from Park College, Parkville, Missouri, with A. B. degree in 1908. Graduate student, as University scholar in Astronomy, at the University of Missouri, 1908-09, and summers of 1909-10; assistant and graduate student 1911-13, receiving the A. M. degree in 1912. Professor of Mathematics, Buena Vista College, Storm Lake, Iowa, 1909-11. Assistant, U. S. Naval Observatory, Washington, D. C., 1913-19. Assigned to testing of nautical instruments 1918-19. Member of U. S. Eclipse Expedition to Baker, Oregon, June 1918. Assistant and graduate student, University of Illinois, February 1920-1922. Married December 25, 1918, to Miss Lirl Bunn, Three Rivers, Michigan.

Magazine articles published include: Determination of the Longitude and Latitude of the U. S. Naval Observatory Eclipse Station at Baker, Oregon, Popular Astronomy, 27, 359, 1919; The Auroral Radiant from American Observations of 1915-1920, Terrestrial Magnetism and Atmospheric Electricity, 25, 163, 1920; The Effect of the Barometric Gradient on Meridian Observations, Popular Astronomy, 29, 479, 1921 (Read at the twenty-fifth meeting of the American Astronomical Society); Elements of 1917 W15, Astronomical Journal, 31, 65, 1918 (with E. C. Bower); Second Elements and Ephemeris of 1917 W15, Astronomical Journal, 31, 65, 1918 (with E.C. Bower).

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